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MODELS OF MORTALITY RATES - ANALYSING THE RESIDUALS

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ABSTRACT. The area of mortality modelling has received significant attention over the last 20 years owing to the need to quantify and forecast improving mortality rates. This need is driven primarily by the concern of governments, professionals, insurance and actuarial professionals and individuals to be able to fund their old age..In particular, to quantify the costs of increasing longevity we need suitable model of mortality rates that capture the dynamics of the data and forecast them with sufficient accuracy to make them useful. In this paper we test several of those models by considering the fitting quality and in particular, testing the residuals of those models for normality properties. In a wide ranging study considering 30 countries we find that almost exclusively the residuals do not demonstrate normality. Further, in hurst tests of the residuals we find evidence that structure remains that is not captured by the models.

JEL Classification: C51, C52, C53, G22, G23, J11

Keywords and Phrases: Mortality, stochastic models, forecasting, structural breaks. hurst exponents

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1. INTRODUCTION

The rapid growth in the development of models of mortality designed to capture patterns in mortality data and accurate forecast and quantify future mortality rates has been dramatic. Over the past recent decades, life expectancy in developed countries has risen to historically unprecedented levels and there is clearly a need from a demographic, financial, social and actuarial perspective to understand and predict these improvements for the future. The prospects of future reductions in mortality rates are of fundamental importance in various areas such as public health and old age care planning, social insurance planning, welfare benefit forecasting and economic policy. Over recent years, significant progress has been made in mortality forecasting (for a recent review see Booth and Tickle, 2008) with the most popular approaches to long-term forecasting being based on the Lee and Carter (1992) model. IA time series model, it describes the movement of age-specific mortality as a function of a latent level of mortality, also known as the overall mortality index, which can be forecasted using simple time-series methods. The method was initially used to forecast mortality in the US, but since then has been applied to many other countries (amongst others see Tuljapurkar and Boe, 1998; Carter and Prskawetz, 2001; Lee and Miller, 2001; Booth *et al.*, 2002; Brouhns *et al.*, 2002; Renshaw and Haberman, 2003 and Koissi *et al.*, 2005).

The success of the Lee Carter model can be seen in the number and variety of mortality models that extend the Lee Carter approach (see O'Hare and Li 2012) for examples of these extensions. One thread of extensions to the Lee Carter model involves including additional latent age and period effects with the objective of better fitting the data, producing a less simplistic correlation structures between ages and capturing the non linear profile of mortality data. This has led for example to the models of Renshaw and Haberman (2003), Cairns, Blake and Dowd (2006,2008, 2009), Plat (2009) and O'Hare and Li (2013) for example.. These models extend the Lee Carter approach by including additional period effects and in some cases cohort effects and improve upon each other by producing better fits to the data and in the main better forecasts. In the literature however, there has been limited attempts to test the fitting of such models. The majority of papers calculate point estimates of the average errors produced between the fitted

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and actual rates using one of several measures (for example root mean square errors, mean average percentage errors etc). There has been very little work looking at the patterns of such errors.

One such paper that considered the shape of the residuals in a range of mortality models is that of Dowd *et al* 2010 where the authors assess the residuals for normality carrying out several tests of the mean, variance and skewness of the residuals. Dowd *et al* 2010 fitted a range of models, primarily the Lee Carter (1992) model and a selection of CBD models to data and then after calculating the in sample forecasts they derived standardised residuals from the forecasts and tested these for normality. Their paper concluded that none of the models considered performed well under these tests. In this paper we extend and modify this work in three ways. Firstly, rather than forecasting and tests the derived residuals we calculate the residuals directly from the fitted models. This will enable us to test the model to ensure that all of the structure of the mortality data is being captured prior to forecasting. Secondly, we extend the work by considering several multi factorial models (namely Plat (2009) and O'Hare and Li (2013)) that were not present in eh prior study. Finally, in addition to the normality tests we calculate hurst exponents for each of the residual time series for each country and gender and comment of the findings therein.

The paper is organized as follows. Section 2 presents a brief review of extrapolative models such as the Lee-Carter model and its extensions. Section 3 discusses the methodology we use to test the residuals for normality. In section 4 we discuss the data we have used in this study and in section 5 we presents the results of our analysis. Finally, section 6 concludes with some ideas for further research.

2. Lee-Carter and its variants

The current leading method for forecasting mortality rates is the stochastic extrapolation approach. In this method data is first transformed (by taking natural logarithms) and then analysed using statistical methods to identify and extract patterns. These patterns are then forecast using well known time series approaches. The resulting forecasts are then used to predict future mortality rates. The first and most well known stochastic mortality model of this type is the Lee and Carter (1992) model. Based on US data the model uses a stochastic, time series framework to identify a single period effect pattern in the natural logarithm of mortality rates. This linear trend over time is extracted and using Box-Jenkins an appropriate ARIMA processes is fitted to the data (a random walk with drift in each case). The random walk with drift is forecast and resulting future mortality rates predicted. Also known as a one factor or one principle component approach the model became a benchmark and underlined a new approach to modelling mortality rates for several reasons: the model has an extremely simple structure and so is very easy to communicate; and the use of the random walk with drift enabled the authors not only to predict the expected future mortality rates but also to visualise the uncertainty associated with the predictions. The **Lee-Carter** model, outlined below includes two age dependent parameters a_x and b_x which respectively represent the intercept and gradient for the log mortality rate at each age and the time or period trend κ_t which is forecast using a random walk with drift:

$$\ln(m_{x,t}) = a_x + b_x \kappa_t + \epsilon_{x,t}, \tag{1}$$

where a_x and b_x are age effects and κ_t is a random period effect.

The model is known to be over parameterised and applying the necessary constraints as in the original Lee and Carter (1992) paper the a_x are given by

$$a_x = \frac{1}{N} \sum_{t=1}^{N} \ln m_{x,t}.$$

In the original paper the bilinear part $b_x \kappa_t$ of the model specification was determined as the first singular component of a singular value decomposition (SVD), with the remaining information from the SVD considered to be part of the error structure. The κ_t were then estimated and refitted to ensure the model mapped onto historic data. Finally the subsequent time series κ_t was used to forecast mortality rates.

Despite the attractiveness of the models simplicity it has several weaknesses. Among many discussions of the Lee-Carter model, Cairns *et al.* (2006, 2009, and 2011) summarized the main disadvantage of the model as having only one factor, resulting in mortality improvements at all ages being perfectly correlated (trivial correlation structure). They also note that for countries where a cohort effect is observed in the past, the model gives a poor fit to historical data. The uncertainty in future death rates is proportional to the average improvement rate b_x which for high ages can lead to this uncertainty being too low, since historical improvement rates have often been lower at high ages. Also, the model can result in a lack of smoothness in the estimated age effect b_x.

Despite the weaknesses of the Lee-Carter model it's simplicity has led to it being taken as a benchmark against which other stochastic mortality models can be assessed. There has been a significant amount of literature developing additions to, or modifications of, the Lee-Carter model. For example Booth *et al.* (2002), Brouhns *et al.* (2002), Lee and Miller (2001), Girosi and King (2005), De Jong and Tickle (2006), Delwarde *et al.* (2007) and Renshaw and Haberman (2003, 2006).

Renshaw and Haberman (2006) modified the Lee-Carter model by simply adding a factor to capture cohort effect. The model does have a much better fit for countries such as the UK where a cohort effect has been identified, however it suffers from a lack of robustness perhaps due to the presence of more than one local maximum in the likelihood function. Among others, for instance Currie (2006) noted that if the model was fitted using data from 1961-2000 then the parameters showed qualitatively different characteristics to those obtained when fitting to data from 1981-2000. Furthermore, as noted by Currie (2006), although the model incorporates the cohort effect, for most of the simulated mortality rates the correlation structure is still trivial with the simulated cohort parameters only being relevant for the higher ages at the far end of the projection. Following this analysis Currie (2006) applied a simplified age-period-cohort model of Clayton and Schifflers (1987) to mortality which removed the robustness problem but at the expense of the fitting quality.

Cairns, Blake and Dowd (2006) observed that for England & Wales and United States data, the fitted cohort effect appeared to have a trend in the year of birth. This suggested that the cohort effect was compensating for the lack of a second age-period effect, as well as trying to capture the cohort effect in the data. This led them to introduce a two factor model (**CBD** model hereafter),

$$\text{logit}(q_{x,t}) = \kappa_t^1 + \kappa_t^2(x - \bar{x}) + \epsilon_{x,t}, \tag{2}$$

where $\bar{x}$ is the mean age in the sample range and (κ_t^1, κ_t^2) are assumed to be a bivariate random walk with drift. The model fits a logistically transformed initial rate of mortality, $q_{x,t}$, using two factors which are both period factors. There is no cohort effect allowed for however, this was rectified in Cairns *et al.* (2009), namely capturing the cohort effect as an additional effect on top of the two age-period effects. The initial rate of mortality can be related to the central rate of mortality $m_{x,t}$ through: $q_{x,t} = 1 - \exp(-m_{x,t})$. All these models have multiple factors resulting in a non-trivial correlation structure which mirrors the reality that improvements in mortality rates are different for different age ranges. A further adaptation was also created allowing for the cohort effect to diminish over time. The main problem with these models arises from the fact that they were designed for higher ages and so ignored the modelling of mortality at the lower ages (for example the accident hump). Cairns *et al.* (2009) argue that the significant cost associated with mortality is at the older ages and thus their modelling focused on those ages. When using these models for full age ranges, the fit quality is relatively poor and the projections are biologically unreasonable.

Plat (2009) wanted to develop a model which maintained the good aspects of the existing models whilst leaving out the weaker features. The result was a four factor model which took its beginnings from the Lee-Carter model and which added factors to capture the second age-period effect, as per the CBD model and the cohort effect, as per the Renshaw and Haberman (2006) model. The innovation in the Plat model was to then add a further period factor affecting only the lower ages and designed to allow the model to fit to the whole age range. The **Plat** model specification is given by:

$$\ln(m_{x,t}) = a_x + \kappa_t^1 + \kappa_t^2(\bar{x} - x) + \kappa_t^3(\bar{x} - x)^+ + \gamma_{t-x} + \epsilon_{x,t}, \tag{3}$$

where the a_x is similar to that of the Lee-Carter model and makes sure that the overall shape of the mortality curve by age is reasonable, the κ_t^1 and κ_t^2 model the mortality rates as in the CBD model and the κ_t^3 models the effects specific to the lower ages only where $(\bar{x} - x)^+$ takes the value $(\bar{x} - x)$ when this is positive and zero otherwise. Finally the γ_{t-x} models the cohort effect.

O'Hare and Li (2012) modified the Plat (2009) model to provide a better fit for a wider age range including ages 5-20. They improved the Plat specification by including a quadratic lower age effect. Their model specification is given below (**OL** model hereafter):

$$\ln(m_{x,t}) = a_x + \kappa_t^1 + \kappa_t^2(\bar{x} - x) + \kappa_t^3\big((\bar{x} - x)^+ + [(\bar{x} - x)^+]^2\big) + \gamma_{t-x} + \epsilon_{x,t}, \tag{4}$$

where a_x makes sure that the basic shape of the mortality curve over ages is in line with historical observations as in the Lee-Carter model (1) and the κ_t^1 factor represents changes in the level of mortality for all ages. Following the reasoning in Cairns *et al.* (2006), the (long-term) stochastic process for this factor should not be mean reverting. The κ_t^2 factor allows changes in mortality to vary between ages reflecting the historical observation that improvement rates can differ for different age classes and κ_t^3 models the effects specific to the lower age only as in the Plat model (3). The adjusted coefficient of κ_t^3 is designed to capture some of the non-linear effects observed at the lower ages, the "quadratic lower age effect".

3. DATA

The data that we use in this paper comes from the Human Mortality Database.[1] The data available for each country includes number of deaths $D_{x,t}$ and exposure to death $E_{x,t}$ for lives aged x last birthday during year t. We can use this to gain a proxy for the central mortality rate for lives aged x during year t as:

$$m_{x,t} = \frac{D_{x,t}}{E_{x,t}}. \tag{5}$$

The data provides an estimate of the true mortality due to issues with the recording of data. Death data tends to be recorded accurately, with death certificates in most cases. However, exposure data is taken from census data which may only be accurately recorded every 5 or 10 years adjusting these figures for migration, deaths and births etc. The resulting mortality estimates are therefore quite noisy, particularly at the older ages where there is less data available. Data is available going back to the mid nineteenth century in some cases but we have restricted this study to data from 1960-2009 in order to have a consistent period across all countries and we have considered the following 30 countries in table 1 in this study.

The wide range of countries give a good spread of populations both geographically and in terms of economic development. The inclusion of Male and Female data also enables gender differences to be considered. We focus on the age range 20-89 for several reasons. Firstly, the models upon which we have based our comparisons are also fitted to this age range. Secondly, and as identified by Currie (2011), data at the older ages provide additional problems in terms

[1]This can be found at http://www.mortality.org/. The database is maintained in the Department of Demography at the University of California, Berkeley, USA, and at the Max Planck Institute for Demographic Research in Rostock, Germany.

TABLE 1. Countries considered in this study along with HMD codes

HMD Code	Country	HMD Code	Country
nor	Norway	pol	Poland
fin	Finland	usa	United States of America
lit	Lithuania	por	Portugal
spa	Spain	ukr	Ukraine
ast	Austria	czr	Czechoslovakia
fra	France	ity	Italy
swe	Sweeden	rus	Russia
blr	Belarus	den	Denmark
nth	Netherlands	jap	Japan
swi	Switzerland	svk	Slovakia
bel	Belgium	est	Estonia
hun	Hungary	lat	Latvia
nzd	New Zealand	ukt	United Kingdom
bul	Bulgaria	uks	Scotland
can	Canada	uke	England

of the reliability. Indeed in several cases mortality rates determined using older data appear to fall sharply beyond age 95.

4. METHODOLOGY

We begin by fitting each of the models considered to the data above for the 30 countries considered and for both males and females. In this paper we will consider the 4 models Lee Carter (1992), CBD(2006), Plat (2009) and O'Hare and Li(2013). We fit the models using a maximum likelihood approach using code developed in R and publicly available for several of the models. The results of fitting are assessed and presented using three point measures of fit quality outlined below.

The average error, $E1$ – this equals the average of the standardized errors,

$$E1 = \frac{1}{X_1 - X_2 + 1} \sum_{x=X_1}^{X_2} \sum_{t=1}^{T} \frac{(\hat{m}_{x,t} - m_{x,t})}{\hat{m}_{x,t}}, \tag{6}$$

this is a measure of the overall bias in the projections. The average absolute error, $E2$ – this equals the average of absolute value of the standardized errors,

$$E2 = \frac{1}{X_1 - X_2 + 1} \sum_{x=X_1}^{X_2} \sum_{t=1}^{T} \left| \frac{\hat{m}_{x,t} - m_{x,t}}{\hat{m}_{x,t}} \right|, \tag{7}$$

this is a measure of the magnitude of the differences between the actual and projected rates. The standard deviation of the error, $E3$ – this equals the square root of the average of the squared errors,

(8) $$E3 = \sqrt{\frac{1}{X_1 - X_2 + 1} \sum_{x=X_1}^{X_2} \sum_{t=1}^{T} \left(\frac{\hat{m}_{x,t} - m_{x,t}}{\hat{m}_{x,t}} \right)^2}.$$

where X_1 and X_2 and the age limits of our sample $X_1 = 20$ and $X_2 = 89$, and $T = 60$ is the number of years of data we have in our sample.

The models are fitted by assuming that death rates are drawn from a poisson distribution with parameter given by $E_{x,t}m_{x,t}$. We then calculate the corresponding fitted mortality rates $\hat{m}_{x,t}$ and calculate the standardised residuals using the following formula

(9) $$\frac{\hat{m}_{x,t} - m_{x,t}}{\sqrt{m_{x,t}/E_{x,t}}}$$

This approach to calculating the residuals is consistent with that of the Dowd *et al* 2010 paper and should represent samples drawn from a standard normal distribution if indeed the residuals are reflecting no more than random noise.The tests used in this section aim to identify whether the mortality residuals described above are consistent with iid N(0,1). We carry out the following tests on the matrix of mortality residuals:

- A t-test of the prediction that their mean should be 0.
- A variance ratio (VR) test of the prediction that the variance should be 1 (see Cochrane, 1988; Lo and MacKinley, 1988, 1989), and
- A JarqueBera normality test based on the skewness and kurtosis predictions (see Jarque and Bera, 1980).

In addition, we calculate hurst exponent, H, for each of the time series extracted from the residuals. The Hurst exponent is referred to as the "index of dependence" or "index of long-range dependence". It quantifies the relative tendency of a time series either to regress strongly to the mean or to cluster in a direction. A value H in the range $0.5 < H < 1$ indicates a time series with long-term positive autocorrelation, meaning both that a high value in the series will probably be followed by another high value and that the values a long time into the future will also tend to be high. A value in the range $0 < H < 0.5$ indicates a time series with long-term switching between high and low values in adjacent pairs, meaning that a single high value will probably be followed by a low value and that the value after that will tend to be high, with this tendency to switch between high and low values lasting a long time into the future. A value of

$H = 0.5$ can indicate a completely uncorrelated series, but in fact it is the value applicable to series for which the autocorrelations at small time lags can be positive or negative but where the absolute values of the autocorrelations decay exponentially quickly to zero. Given that we are expecting the residuals to be samples for a N(0,1) distribution we should not expect any corelations between residuals. In other words a husrt exponent of 0,5 would be ideal.

The Hurst exponent, H, is defined in terms of the asymptotic behaviour of the rescaled range as a function of the time span of a time series as follows

$$E\left[\frac{R(n)}{S(n)}\right] = Cn^H\_\text{as}\_n \to \infty \tag{10}$$

where;

- $R(n)$ is the range of the first n values, and $S(n)$ is their standard deviation
- $E[\_]$ is the expected value
- n is the time span of the observation (number of data points in a time series)
- C is a constant

In order to consider the hurst exponent analysis we must apply it to a time series of residuals not a matrix of residuals. We therefore consider both the age dimension and the period dimension separately. In both cases we should not expect any correlations between residuals across age nor should we expect any across the period dimension. In the empirical section following we present the analysis in both dimensions and comment accordingly.

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5. Empirical analysis

In this action we present and discuss our findings. We firstly show the fitting results measured using the standard E1, E2 and E3 measures of fitting quality. These are calculated as shown in the methodology section and in the main confirm the reported findings of each of the previous papers proposing the models. We follow this with a discussion of some of the residuals calculated for each of the countries in the study. We present some of the residual plots and comment on some common characteristics we find. Finally, we empirically test the residuals using a range of tests as discussed above.

5.1. **Fitting the models and assessing with point estimates.** We consider each of the 30 countries covered in the paper for both male and female data, fitting the models to data from 1960 - 2009. We present the results in tables including all countries but where we only populate the cases where a structural break was present. We present results below in tables 4 - **??** using the three measures of error E1, E2 and E3 outlines earlier.

TABLE 2. Fitting results (expressed as percentages) measured using the mean absolute percentage error (E1) for Males and Females for the Lee-Carter, CBD, Plat, and OL models.

Country	Male				Female			
	LC(1992)	CBD(2006)	Plat(2009)	OL(2013)	LC(1992)	CBD(2006)	Plat(2009)	OL(2013)
Norway	7.18	15.41	6.62	6.79	9.54	20.24	9.56	9.77
Finland	7.48	11.63	6.4	6.34	10.9	22.56	10.36	10.73
Lithuania	8.67	11.27	6.93	7.08	11.41	19.27	10	10.17
Spain	7.65	14.4	4.49	4.68	6.7	26.24	5.45	5.67
Austria	6.39	14.11	5.83	5.91	7.96	23.1	7.9	8.23
France	5.3	12.38	3.06	3.33	5.04	26.54	3.76	4.19
Sweeden	6.3	15.86	6.17	6.04	8.55	21.86	8.39	8.6
Belarus	6.51	8.55	5.04	4.96	8.33	15.58	7.24	7.45
Netherlands	5.69	13.74	4.38	4.47	5.81	18.85	5.6	6.01
Switzerland	8.73	18.31	6.71	6.69	10.46	24.33	9.41	9.5
Belgium	6.27	14.87	4.93	4.94	7.52	22.24	6.92	7.26
Hungary	10.59	13.97	5.93	5.24	8.46	15.65	6.38	6.21
New Zeland	9.52	19.66	9.35	9.52	12.14	16.41	12.2	12.09
Bulgaria	6.75	11.58	4.88	5.42	7.77	19.45	6.92	7.1
Canada	4.83	14.81	3.98	4.14	5	12.52	4.44	4.48
Poland	6.93	9.45	3.1	3.37	5.13	15.1	3.98	4.68
USA	4.46	13.05	2.65	2.67	3.85	11.81	3.01	3
Portugal	8.51	17.69	6.1	5.83	7.05	25.99	7.17	7.28
Ukraine	5.09	9.01	4.3	4.18	7.84	17.63	5.08	5.42
Czechoslovakia	6.46	11.28	4.81	4.82	7.15	15.48	6.6	6.93
Italy	6.31	14.06	3.78	3.93	5.3	21.83	4.06	4.46
Russia	5.03	8.71	3.7	3.72	6.48	17.53	4.16	4.42
Denmark	8.02	12.52	7.13	6.98	12.92	14.13	9.93	10.04
Japan	4.86	14.09	3.45	3.78	6.84	25.02	2.94	3.57
Slovakia	10	11.83	6.19	6.13	10.61	15.51	9.89	9.85
Estonia	11.66	13.21	9.76	9.86	18.55	20.93	16.83	16.91
Latvia	9.54	11.4	6.98	7.16	12.1	19.04	10.2	10.07
United Kingdom	4.74	14.87	3.76	4.1	4.88	11.78	3.45	3.7
Scotland	7.88	15.06	7.45	7.92	9.98	11.48	8.76	8.84
England	4.94	15.05	3.9	4.2	4.98	12.37	3.62	3.86

TABLE 3. Fitting results (expressed as percentages) measured using the mean average percentage error (E2) for Males and Females for the Lee-Carter, CBD, Plat, and OL models.

Country	Male				Female			
	LC(1992)	CBD(2006)	Plat(2009)	OL(2013)	LC(1992)	CBD(2006)	Plat(2009)	OL(2013)
Norway	0.98	-5.22	0.87	0.95	2.04	-12.73	2.04	2.04
Finland	1.13	-3.3	0.74	0.79	2.65	-12.41	2.27	2.27
Lithuania	1.27	-1.37	0.85	0.79	2.58	-8.32	2.16	2.01
Spain	2.15	-5.92	0.62	0.45	1.33	-15.05	0.89	0.65
Austria	0.9	-4.45	0.52	0.57	1.47	-14.51	1.09	1.16
France	1.07	-4.7	0.3	0.3	0.87	-13.97	0.39	0.24
Sweeden	0.73	-8.47	0.69	0.76	1.72	-14.29	1.24	1.39
Belarus	0.85	-1.17	0.49	0.46	1.16	-7.84	1.08	1.09
Netherlands	0.22	-2.43	0.45	0.44	0.97	-11.07	0.68	0.54
Switzerland	2.09	-5.24	1.09	1.02	2.93	-13.92	1.98	1.91
Belgium	0.9	-4.25	0.57	0.6	1.38	-13.37	1.02	0.99
Hungary	2.16	2.51	0.93	0.25	1.63	-7.04	1.23	0.59
New Zeland	1.89	-1.05	1.73	1.73	2.94	-5.26	2.97	2.95
Bulgaria	0.91	-2.81	0.57	0.47	1.39	-10.99	1.16	1.08
Canada	0.48	-2.68	0.48	0.49	0.43	-6.86	0.41	0.45
Poland	1.03	-1.7	0.31	0.16	0.56	-8.35	0.4	0.28
USA	0.69	-2.67	0.31	0.33	0.23	-6.45	0.26	0.25
Portugal	2.25	-6.58	1.08	0.76	1.13	-14.08	1.14	1.01
Ukraine	0.63	-2.3	0.56	0.45	1.19	-9.91	0.68	0.68
Czechoslovakia	0.92	-0.69	0.46	0.37	1.23	-9.92	0.92	0.89
Italy	1.05	-4.84	0.47	0.51	0.7	-14.31	0.39	0.35
Russia	0.59	-2.08	0.39	0.41	0.7	-9.18	0.35	0.37
Denmark	1.36	-3.02	1.11	1.02	3.31	-4.26	2.21	2.18
Japan	0.42	-7.02	0.3	0.47	0.32	-13.99	0.28	0.33
Slovakia	1.27	-0.25	0.76	0.54	2.4	-7.09	2.22	2.09
Estonia	2.05	-0.76	1.82	1.88	6.95	-7.53	6.18	6.06
Latvia	1.99	-1.27	0.9	0.81	3.03	-9.35	2.52	2.19
United Kingdom	0.46	-1.68	0.55	0.64	0.33	-7.75	0.3	0.37
Scotland	1.27	0.11	1.38	1.52	1.97	-2.89	1.89	2
England	0.5	-1.76	0.57	0.65	0.37	-8.2	0.34	0.41

TABLE 4. Fitting results (expressed as percentages) measured using the root mean square percentage error (E3) for Males and Females for the Lee-Carter, CBD, Plat, and OL models.

Country	Male				Female			
	LC(1992)	CBD(2006)	Plat(2009)	OL(2013)	LC(1992)	CBD(2006)	Plat(2009)	OL(2013)
Norway	0.88	1.47	0.72	0.75	0.68	1.58	1.33	1.1
Finland	1.37	1.77	1.27	1.29	0.97	1.87	2.76	2.19
Lithuania	1.76	2.47	1.54	1.46	1.39	2.31	1.97	1.57
Spain	0.72	1.81	1.27	1.08	0.55	1.95	2.22	1.82
Austria	1.01	1.86	1.36	1.17	0.71	1.9	1.79	1.35
France	0.67	2.15	1.39	1.15	0.49	2.37	1.8	1.37
Sweeden	0.74	1.44	0.86	0.81	0.61	1.66	1.3	1.02
Belarus	1.44	1.95	1.17	1.18	1.28	1.87	1.22	1.15
Netherlands	0.92	1.16	0.49	0.5	0.48	1.72	1.52	1.17
Switzerland	0.97	1.95	1.15	1.1	0.74	2.1	1.87	1.54
Belgium	0.94	1.63	1.01	0.97	0.62	1.93	1.87	1.46
Hungary	2.64	1.88	1.57	1.07	0.92	1.82	1.83	1.34
New Zeland	1.47	2.07	1.45	1.51	1.19	1.8	1.25	1.22
Bulgaria	1.62	1.82	1.8	1.39	1.41	1.78	1.74	1.48
Canada	0.78	1.35	0.59	0.66	0.52	1.37	0.58	0.55
Poland	1.27	1.59	1.2	0.89	0.67	1.69	1.48	1.02
USA	0.64	1.68	0.72	0.77	0.44	1.49	0.46	0.45
Portugal	0.99	2.59	1.6	1.39	0.69	2.08	1.79	1.51
Ukraine	1.29	2.4	1.23	1.13	1.09	2.02	1.13	1.03
Czechoslovakia	1.1	1.37	1.12	1.02	0.72	1.34	1.21	0.96
Italy	0.79	1.41	0.49	0.5	0.53	1.67	1.26	0.99
Russia	1.38	2.49	1.17	1.16	1.16	2.23	1.22	1.08
Denmark	1.14	1.23	1.01	0.97	1.02	1.55	0.85	0.78
Japan	0.73	1.86	1.3	1.02	0.91	2.23	1.83	1.42
Slovakia	2.17	2.13	1.17	1.15	1.16	1.72	1.2	1.07
Estonia	2.8	2.86	2.18	2.13	1.57	2.19	1.72	1.53
Latvia	1.86	2.61	1.53	1.53	1.22	2.18	1.38	1.18
United Kingdom	0.65	1.44	0.77	0.95	0.5	0.98	0.63	0.57
Scotland	1.07	1.68	0.96	1.07	0.89	1.04	0.8	0.8
England	0.67	1.46	0.83	1	0.5	1.01	0.65	0.59

5.2. **Analysing the residuals.**

5.3. **Statistical tests of the residuals.**

5.3.1. *t-tests of the mean.*

TABLE 5. t-test results both statistic and p-value for the Lee-Carter, CBD, Plat, and OL models.

Country	Lee-Carter				CBD				Plat				OL			
	Male		Female		Male		Female		Male		Female		Male		Female	
	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
Norway	0.586	0.000	0.730	0.000	0.607	0.000	0.753	0.000	0.614	0.000	0.121	0.000	0.567	0.000	0.175	0.000
Finland	0.400	0.000	0.513	0.000	0.461	0.000	0.711	0.000	0.266	0.000	0.066	0.000	0.264	0.000	0.086	0.000
Lithuania	0.565	0.000	0.537	0.000	0.544	0.000	0.708	0.000	0.378	0.000	0.118	0.000	0.534	0.000	0.181	0.000
Spain	0.559	0.000	0.516	0.000	0.582	0.000	0.749	0.000	0.079	0.000	0.053	0.000	0.099	0.000	0.064	0.000
Austria	0.559	0.000	0.643	0.000	0.601	0.000	0.760	0.000	0.158	0.000	0.076	0.000	0.218	0.000	0.111	0.000
France	0.593	0.000	0.679	0.000	0.551	0.000	0.728	0.000	0.068	0.000	0.050	0.000	0.090	0.000	0.068	0.000
Sweeden	0.493	0.000	0.600	0.000	0.652	0.000	0.750	0.000	0.234	0.000	0.105	0.000	0.276	0.000	0.165	0.000
Belarus	0.535	0.000	0.560	0.000	0.560	0.000	0.668	0.000	0.454	0.000	0.260	0.000	0.510	0.000	0.340	0.000
Netherlands	0.520	0.000	0.553	0.000	0.588	0.000	0.670	0.000	0.635	0.000	0.061	0.000	0.603	0.000	0.087	0.000
Switzerland	0.402	0.000	0.498	0.000	0.552	0.000	0.680	0.000	0.182	0.000	0.080	0.000	0.204	0.000	0.103	0.000
Belgium	0.597	0.000	0.726	0.000	0.635	0.000	0.752	0.000	0.190	0.000	0.070	0.000	0.208	0.000	0.092	0.000
Hungary	0.551	0.000	0.587	0.000	0.554	0.000	0.665	0.000	0.129	0.000	0.067	0.000	0.214	0.000	0.089	0.000
New Zeland	0.494	0.000	0.652	0.000	0.624	0.000	0.641	0.000	0.461	0.000	0.431	0.000	0.420	0.000	0.500	0.000
Bulgaria	0.537	0.000	0.392	0.000	0.563	0.000	0.674	0.000	0.109	0.000	0.092	0.000	0.183	0.000	0.118	0.000
Canada	0.589	0.000	0.550	0.000	0.635	0.000	0.562	0.000	0.357	0.000	0.211	0.000	0.266	0.000	0.259	0.000
Poland	0.553	0.000	0.413	0.000	0.512	0.000	0.606	0.000	0.112	0.000	0.060	0.000	0.211	0.000	0.097	0.000
USA	0.622	0.000	0.606	0.000	0.578	0.000	0.554	0.000	0.117	0.000	0.248	0.000	0.112	0.000	0.340	0.000
Portugal	0.493	0.000	0.641	0.000	0.573	0.000	0.765	0.000	0.123	0.000	0.094	0.000	0.159	0.000	0.114	0.000
Ukraine	0.503	0.000	0.541	0.000	0.500	0.000	0.640	0.000	0.206	0.000	0.172	0.000	0.329	0.000	0.237	0.000
Czechoslovakia	0.512	0.000	0.635	0.000	0.553	0.000	0.720	0.000	0.256	0.000	0.110	0.000	0.339	0.000	0.175	0.000
Italy	0.493	0.000	0.527	0.000	0.468	0.000	0.651	0.000	0.482	0.000	0.060	0.000	0.443	0.000	0.088	0.000
Russia	0.516	0.000	0.454	0.000	0.512	0.000	0.658	0.000	0.255	0.000	0.137	0.000	0.353	0.000	0.201	0.000
Denmark	0.475	0.000	0.629	0.000	0.630	0.000	0.560	0.000	0.337	0.000	0.335	0.000	0.372	0.000	0.532	0.000
Japan	0.303	0.000	0.290	0.000	0.511	0.000	0.524	0.000	0.055	0.000	0.042	0.000	0.085	0.000	0.056	0.000
Slovakia	0.423	0.000	0.483	0.000	0.444	0.000	0.562	0.000	0.448	0.000	0.214	0.000	0.465	0.000	0.283	0.000
Estonia	0.544	0.000	0.686	0.000	0.471	0.000	0.691	0.000	0.397	0.000	0.299	0.000	0.455	0.000	0.463	0.000
Latvia	0.592	0.000	0.717	0.000	0.519	0.000	0.669	0.000	0.487	0.000	0.272	0.000	0.554	0.000	0.431	0.000
United Kingdom Total	0.551	0.000	0.694	0.000	0.552	0.000	0.748	0.000	0.144	0.000	0.137	0.000	0.120	0.000	0.192	0.000
Scotland	0.572	0.000	0.748	0.000	0.558	0.000	0.715	0.000	0.547	0.000	0.491	0.000	0.449	0.000	0.531	0.000
England	0.552	0.000	0.690	0.000	0.544	0.000	0.756	0.000	0.136	0.000	0.136	0.000	0.115	0.000	0.190	0.000

5.3.2. *variance ratio tests.*

TABLE 6. Variance ratio test results (x 10^5) for the Lee-Carter, CBD, Plat, and OL models.

Country	Lee-Carter				CBD				Plat				OL			
	Male		Female		Male		Female		Male		Female		Male		Female	
	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
Norway	0.002	0.000	0.001	0.000	0.005	0.000	0.003	0.000	0.001	0.000	0.009	0.000	0.001	0.000	0.005	0.000
Finland	0.014	0.000	0.002	0.000	0.017	0.000	0.005	0.000	0.019	0.000	0.073	0.000	0.021	0.000	0.045	0.000
Lithuania	0.018	0.000	0.006	0.000	0.035	0.000	0.010	0.000	0.022	0.000	0.033	0.000	0.013	0.000	0.017	0.000
Spain	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.003	0.000	0.004	0.000	0.002	0.000	0.003	0.000
Austria	0.002	0.000	0.000	0.000	0.006	0.000	0.002	0.000	0.011	0.000	0.011	0.000	0.006	0.000	0.006	0.000
France	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.002	0.000	0.001	0.000	0.001	0.000	0.001	0.000
Sweeden	0.001	0.000	0.000	0.000	0.002	0.000	0.002	0.000	0.002	0.000	0.005	0.000	0.002	0.000	0.002	0.000
Belarus	0.005	0.000	0.002	0.000	0.010	0.000	0.003	0.000	0.004	0.000	0.003	0.000	0.004	0.000	0.002	0.000
Netherlands	0.001	0.000	0.000	0.000	0.001	0.000	0.002	0.000	0.000	0.000	0.007	0.000	0.000	0.000	0.004	0.000
Switzerland	0.003	0.000	0.001	0.000	0.007	0.000	0.004	0.000	0.008	0.000	0.015	0.000	0.007	0.000	0.010	0.000
Belgium	0.001	0.000	0.000	0.000	0.002	0.000	0.002	0.000	0.003	0.000	0.008	0.000	0.003	0.000	0.004	0.000
Hungary	0.013	0.000	0.001	0.000	0.006	0.000	0.003	0.000	0.015	0.000	0.016	0.000	0.005	0.000	0.008	0.000
New Zeland	0.015	0.000	0.004	0.000	0.020	0.000	0.010	0.000	0.017	0.000	0.007	0.000	0.019	0.000	0.006	0.000
Bulgaria	0.005	0.000	0.005	0.000	0.006	0.000	0.003	0.000	0.023	0.000	0.017	0.000	0.010	0.000	0.011	0.000
Canada	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
Poland	0.001	0.000	0.000	0.000	0.002	0.000	0.001	0.000	0.006	0.000	0.004	0.000	0.002	0.000	0.002	0.000
USA	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Portugal	0.002	0.000	0.000	0.000	0.011	0.000	0.002	0.000	0.017	0.000	0.009	0.000	0.012	0.000	0.006	0.000
Ukraine	0.001	0.000	0.000	0.000	0.004	0.000	0.001	0.000	0.002	0.000	0.001	0.000	0.001	0.000	0.001	0.000
Czechoslovakia	0.003	0.000	0.000	0.000	0.003	0.000	0.001	0.000	0.006	0.000	0.005	0.000	0.004	0.000	0.003	0.000
Italy	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.000
Russia	0.001	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000
Denmark	0.004	0.000	0.001	0.000	0.003	0.000	0.004	0.000	0.004	0.000	0.002	0.000	0.003	0.000	0.001	0.000
Japan	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.002	0.000	0.002	0.000	0.001	0.000	0.001	0.000
Slovakia	0.038	0.000	0.005	0.000	0.033	0.000	0.009	0.000	0.010	0.000	0.011	0.000	0.008	0.000	0.007	0.000
Estonia	0.143	0.000	0.010	0.000	0.159	0.000	0.019	0.000	0.109	0.000	0.027	0.000	0.091	0.000	0.014	0.000
Latvia	0.025	0.000	0.003	0.000	0.061	0.000	0.012	0.000	0.022	0.000	0.011	0.000	0.020	0.000	0.005	0.000
United Kingdom Total	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
Scotland	0.003	0.000	0.001	0.000	0.009	0.000	0.001	0.000	0.003	0.000	0.001	0.000	0.004	0.000	0.001	0.000
England	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000

5.3.3. *Jaque Bara tests.*

TABLE 7. Jaque Bera test results (x $10^{-}5$) for Males and Females for the Lee-Carter, CBD, Plat, and OL models.

Country	Lee-Carter				CBD				Plat				OL			
	Male		Female		Male		Female		Male		Female		Male		Female	
	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
Norway	3.087	0.000	1.389	0.000	1.971	0.000	0.220	0.000	23.250	0.000	263.228	0.000	46.011	0.000	227.961	0.000
Finland	13.900	0.000	8.053	0.000	5.985	0.000	0.437	0.000	106.457	0.000	285.736	0.000	95.226	0.000	267.855	0.000
Lithuania	2.258	0.000	4.709	0.000	1.167	0.000	0.334	0.000	41.164	0.000	551.076	0.000	3.982	0.000	469.414	0.000
Spain	3.312	0.000	4.749	0.000	0.382	0.000	0.100	0.000	278.679	0.000	317.274	0.000	263.264	0.000	307.531	0.000
Austria	7.803	0.000	3.954	0.000	0.564	0.000	0.153	0.000	377.325	0.000	328.142	0.000	315.070	0.000	309.824	0.000
France	3.132	0.000	1.413	0.000	0.349	0.000	0.156	0.000	274.540	0.000	305.675	0.000	254.130	0.000	291.958	0.000
Sweeden	23.095	0.000	14.099	0.000	0.754	0.000	0.170	0.000	141.512	0.000	243.880	0.000	114.864	0.000	188.423	0.000
Belarus	5.023	0.000	2.584	0.000	0.798	0.000	0.546	0.000	31.451	0.000	283.186	0.000	6.841	0.000	172.854	0.000
Netherlands	5.110	0.000	9.879	0.000	2.182	0.000	0.217	0.000	1.091	0.000	290.964	0.000	2.669	0.000	271.957	0.000
Switzerland	30.901	0.000	41.375	0.000	2.068	0.000	0.530	0.000	186.525	0.000	331.420	0.000	159.932	0.000	316.360	0.000
Belgium	3.334	0.000	0.875	0.000	0.854	0.000	0.165	0.000	160.927	0.000	291.784	0.000	147.883	0.000	273.412	0.000
Hungary	1.858	0.000	6.289	0.000	1.297	0.000	0.360	0.000	244.568	0.000	275.198	0.000	187.756	0.000	265.453	0.000
New Zeland	4.246	0.000	2.155	0.000	1.594	0.000	0.953	0.000	8.041	0.000	60.703	0.000	14.340	0.000	29.083	0.000
Bulgaria	5.381	0.000	13.531	0.000	2.281	0.000	1.358	0.000	203.088	0.000	224.766	0.000	138.037	0.000	196.132	0.000
Canada	1.714	0.000	4.558	0.000	0.537	0.000	0.595	0.000	51.566	0.000	173.587	0.000	102.217	0.000	138.133	0.000
Poland	3.158	0.000	17.505	0.000	1.096	0.000	0.420	0.000	215.967	0.000	257.179	0.000	109.650	0.000	225.644	0.000
USA	3.455	0.000	3.352	0.000	0.541	0.000	0.633	0.000	198.493	0.000	151.272	0.000	203.701	0.000	72.157	0.000
Portugal	10.977	0.000	2.631	0.000	0.541	0.000	0.268	0.000	242.818	0.000	271.889	0.000	196.130	0.000	262.347	0.000
Ukraine	19.308	0.000	17.496	0.000	0.517	0.000	0.570	0.000	339.572	0.000	345.090	0.000	103.748	0.000	250.368	0.000
Czechoslovakia	6.577	0.000	1.288	0.000	2.445	0.000	0.311	0.000	74.479	0.000	276.869	0.000	19.672	0.000	200.022	0.000
Italy	10.987	0.000	9.422	0.000	5.215	0.000	0.506	0.000	5.256	0.000	295.813	0.000	6.433	0.000	270.590	0.000
Russia	3.875	0.000	36.616	0.000	0.300	0.000	0.276	0.000	128.581	0.000	351.299	0.000	25.257	0.000	255.018	0.000
Denmark	6.933	0.000	17.213	0.000	1.951	0.000	2.143	0.000	80.912	0.000	125.165	0.000	66.972	0.000	27.150	0.000
Japan	59.494	0.000	21.161	0.000	0.775	0.000	0.792	0.000	319.228	0.000	305.478	0.000	301.226	0.000	289.400	0.000
Slovakia	8.875	0.000	24.070	0.000	4.478	0.000	2.298	0.000	6.106	0.000	337.755	0.000	4.207	0.000	280.689	0.000
Estonia	2.523	0.000	2.101	0.000	2.960	0.000	0.738	0.000	17.838	0.000	180.934	0.000	6.878	0.000	57.610	0.000
Latvia	1.988	0.000	1.205	0.000	1.008	0.000	0.377	0.000	8.613	0.000	112.954	0.000	3.082	0.000	34.364	0.000
United Kingdom Total	9.679	0.000	2.616	0.000	1.114	0.000	0.160	0.000	206.389	0.000	258.310	0.000	224.596	0.000	197.558	0.000
Scotland	2.160	0.000	1.234	0.000	1.544	0.000	0.761	0.000	2.249	0.000	34.794	0.000	8.245	0.000	21.795	0.000
England	8.668	0.000	3.185	0.000	1.269	0.000	0.154	0.000	228.038	0.000	263.383	0.000	242.544	0.000	204.847	0.000

5.3.4. *Hurst exponent tests.* The hurst exponent calculations are done by first splitting the matrix of residuals into time series of age specific residuals and period specific residuals. In other words by considering the columns and rows of the matrix separately. Of course we might also consider the cohort pattern (or the diagonals) of the matrix also but we defer this to further study. The results presented below should the hurst exponents over period and over age.

TABLE 8. Hurst exponents for age specific residuals for the Lee-Carter, CBD, Plat, and OL models.

Country	Lee-Carter		CBD		Plat		OL	
	Male	Female	Male	Female	Male	Female	Male	Female
Norway	0.618	0.586	0.709	0.662	0.584	0.589	0.592	0.598
Finland	0.606	0.573	0.666	0.660	0.550	0.533	0.548	0.566
Lithuania	0.590	0.592	0.681	0.660	0.543	0.543	0.559	0.563
Spain	0.676	0.647	0.731	0.720	0.615	0.600	0.632	0.618
Austria	0.577	0.557	0.706	0.674	0.593	0.552	0.599	0.576
France	0.655	0.655	0.719	0.690	0.631	0.616	0.657	0.648
Sweeden	0.573	0.548	0.721	0.661	0.570	0.571	0.567	0.592
Belarus	0.621	0.570	0.662	0.672	0.571	0.572	0.572	0.583
Netherlands	0.644	0.594	0.712	0.669	0.578	0.572	0.583	0.611
Switzerland	0.610	0.550	0.716	0.675	0.558	0.559	0.558	0.565
Belgium	0.615	0.577	0.708	0.677	0.551	0.541	0.551	0.575
Hungary	0.681	0.616	0.700	0.693	0.625	0.583	0.593	0.571
New Zeland	0.585	0.573	0.676	0.644	0.569	0.553	0.571	0.553
Bulgaria	0.606	0.574	0.689	0.678	0.543	0.541	0.591	0.567
Canada	0.662	0.589	0.717	0.693	0.610	0.563	0.614	0.572
Poland	0.693	0.629	0.719	0.709	0.622	0.586	0.634	0.618
USA	0.713	0.692	0.730	0.706	0.653	0.670	0.653	0.674
Portugal	0.653	0.556	0.719	0.704	0.613	0.572	0.621	0.578
Ukraine	0.620	0.605	0.695	0.709	0.581	0.577	0.601	0.600
Czechoslovakia	0.617	0.555	0.671	0.670	0.598	0.544	0.594	0.579
Italy	0.682	0.627	0.723	0.709	0.651	0.626	0.647	0.648
Russia	0.647	0.628	0.709	0.722	0.625	0.619	0.636	0.634
Denmark	0.602	0.616	0.659	0.647	0.547	0.526	0.540	0.542
Japan	0.683	0.645	0.731	0.708	0.669	0.622	0.673	0.647
Slovakia	0.644	0.552	0.679	0.630	0.548	0.518	0.542	0.526
Estonia	0.626	0.549	0.657	0.633	0.556	0.520	0.566	0.530
Latvia	0.618	0.589	0.687	0.668	0.563	0.545	0.585	0.556
United Kingdom	0.670	0.660	0.724	0.693	0.644	0.617	0.644	0.630
Scotland	0.579	0.598	0.698	0.630	0.578	0.544	0.594	0.547
England	0.671	0.653	0.723	0.690	0.639	0.611	0.638	0.625

TABLE 9. Hurst exponents for period specific residuals for the Lee-Carter, CBD, Plat, and OL models.

Country	Lee-Carter		CBD		Plat		OL	
	Male	Female	Male	Female	Male	Female	Male	Female
Norway	0.677	0.580	0.949	0.906	0.795	0.800	0.726	0.847
Finland	0.747	0.782	1.237	0.906	0.696	0.699	0.639	0.914
Lithuania	0.750	0.689	1.138	0.930	0.643	0.683	0.755	0.899
Spain	0.899	0.851	0.970	0.875	0.770	0.651	0.743	0.782
Austria	0.677	0.621	0.896	0.972	0.877	0.867	0.932	0.932
France	0.706	0.916	0.876	0.932	0.782	0.944	0.767	0.977
Sweeden	0.550	0.759	0.787	1.061	0.791	0.986	0.773	0.970
Belarus	0.698	0.621	1.088	0.771	0.765	0.850	0.863	0.925
Netherlands	0.954	0.664	1.198	0.979	0.603	0.832	0.603	0.996
Switzerland	0.769	0.710	1.174	0.716	0.788	0.852	0.776	0.835
Belgium	0.721	0.762	1.307	0.911	0.696	0.806	0.719	0.992
Hungary	0.862	0.927	1.113	1.077	0.895	0.779	0.647	0.754
New Zeland	0.713	0.653	1.075	0.802	0.775	0.877	0.735	0.787
Bulgaria	0.720	0.696	1.195	0.988	0.749	0.653	0.904	0.836
Canada	0.761	0.686	1.223	0.935	0.869	0.786	0.889	0.799
Poland	1.143	0.729	1.196	1.089	0.925	0.960	0.935	1.026
USA	0.804	0.810	1.106	0.992	0.868	0.952	0.863	0.934
Portugal	0.757	0.481	0.839	0.889	0.781	0.637	0.783	0.808
Ukraine	0.847	0.854	1.128	0.914	0.755	0.965	0.838	0.969
Czechoslovakia	0.813	0.665	0.939	0.812	0.856	0.781	0.921	0.957
Italy	0.608	0.784	0.859	0.827	0.955	0.971	0.884	1.006
Russia	0.862	0.755	1.016	0.731	0.795	0.890	0.892	0.954
Denmark	0.717	0.997	0.971	1.106	0.614	0.637	0.591	0.660
Japan	0.819	0.997	1.006	1.089	0.992	0.937	1.025	1.004
Slovakia	1.058	0.712	1.254	1.007	0.772	0.664	0.795	0.718
Estonia	0.928	0.730	1.073	0.725	0.731	0.617	0.729	0.691
Latvia	0.965	0.828	1.177	0.924	0.680	0.670	0.792	0.779
United Kingdom Total	0.807	0.915	1.147	0.937	0.848	1.037	0.871	1.040
Scotland	0.734	0.895	1.214	1.017	0.774	0.808	0.771	0.825
England	0.791	0.895	1.140	0.916	0.837	1.002	0.879	1.023

As can be seen for the test results in every case the fitted models fail the basic normality tests suggesting that the residuals mean and variances do not conform to those of the standard normal distribution, nor do the higher moments. In addition, the hurst exponent calculations show long term positive correlation in the residuals both in the age dimension and the period dimension. This suggests that perhaps there is still some structure in the residuals that might be identified. In particular the inclusion of additional period effects (as in the Plat (2009) and Ohare and Li(2013) models does not compensate for this. This is an area of further research.

6. Conclusions

In this paper we have considered several of the leading extrapolative models of mortality rates and have applied normality tests and hurst calculations to the fitted residuals. More specifically we have fitted the models of Lee and Carter (1992), Cairns, Blake and Dowd (2006), Plat (2009) and O'Hare and Li (2012) to the data for 30 countries for both males and females and tests the resulting residuals using t-tests, variance ratio tests and Jaque bear tests. We have also calculated age and prior hurst exponents for each of the countries and genders and note that exclusively these hurst exponents lie in the region $0.5 < 1$. This suggests some positive correlations between residuals.

Further research will now focus further on the hurst exponents analysing in more detail the patterns found within these exponents to try to identify what if any structure still remains in the data after fitting such time series model.

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